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Structure-Preserving Coupled Field-Cable Simulations via the Port-Hamiltonian Framework

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***Abstract*—A discrete structure-preserving scheme for the stable co-simulation of transmission lines and electromagnetic fields is developed within the port-Hamiltonian framework. The formulation couples Maxwell's equations with telegrapher's equations using mimetic and symplectic discretizations for both subsystems. This approach guarantees passivity, discrete global energy balance, and stability in long-term simulations. Numerical results verify the accuracy of the method and its exact energy-consistent behavior.**

***Index Terms*—port-Hamiltonian systems, structure-preserving discretization, line-field coupling, computational electromagnetics**

I. Introduction

Guided-wave structures embedded in complex electromagnetic environments are central to signal integrity and electromagnetic compatibility studies [1]. In simulations of such systems, fast guided waves and radiating fields need to be resolved in multiple scales, restricting the computational feasibility of full-wave discretizations, while line models alone do not account for field interactions. Hence, robust modeling schemes that preserve physicality and ensure stable, energy-consistent long-term simulations are required.

Standard co-simulation or hybrid schemes often use ad-hoc coupling conditions that lack a physically consistent power balance. Such conditions can break reciprocity, causing spurious reflections, while they often require artificial dissipation or low-order time-staggering for stability [2]. These modifications alter true energy flow, compromise passivity, and can distort wave propagation, leading to phase errors, damping, and numerical instabilities in long-term simulations.

Here, we develop a discrete structure-preserving field-line system within the port-Hamiltonian (pH) framework [3], [4]. By expressing both the telegrapher and Maxwell equations as pH systems, we achieve a power-conserving geometric coupling that preserves the combined system's intrinsic Dirac structure.

II. Problem Formulation

The electromagnetic coupling between a one-dimensional transmission line and a three-dimensional field domain is formulated as the interconnection of two distributed-parameter pH systems; namely, the telegrapher and Maxwell equations. The computational setting for an asymptotic DC problem, with $\ell = 0.5\,\mathrm{m}$, $V_+ = 1\,\mathrm{V}$, and $V_- = 0\,\mathrm{V}$ is shown in Fig. 1.

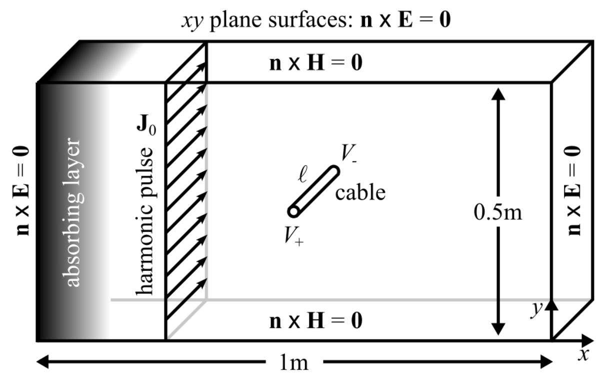

Fig. 1. Setting for the coupled field-line problem.

A. Telegrapher's equations

Let I and V be the current and voltage along a line with per-unit-length resistance $R\,(\Omega/\mathrm{m})$, inductance $L\,(\mathrm{H/m})$, capacitance $C\,(\mathrm{F/m})$, and conductance $G\,(\mathrm{S/m})$. In its pH form, the dynamics in the interior of the line is governed by

$$\begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix} \begin{bmatrix} \dot{I} \\ \dot{V} \end{bmatrix} = \left(\begin{bmatrix} 0 & -\partial_x \\ -\partial_x & 0 \end{bmatrix} - \begin{bmatrix} R & 0 \\ 0 & G \end{bmatrix} \right) \begin{bmatrix} I \\ V \end{bmatrix},$$

where $\partial_x = \partial/\partial x$ and overdots denote partial differentiation with respect to time. The Dirichlet boundary data $V_\pm$ enter the formulation as a boundary power port, $P_{\ell,\partial} = (IV)|_0^\ell$.

B. Maxwell's equations

Let $\Omega \subset \mathbb{R}^3$ be an open, bounded, and simply-connected Lipschitz domain, occupied by a composite of perfect and Ohmic material, with permittivity $\varepsilon\,(\mathrm{F/m})$, permeability $\mu\,(\mathrm{H/m})$, and conductivity $\sigma\,(\mathrm{S/m})$. The electric and magnetic field intensities $\mathbf{E}$ and $\mathbf{H}$ are governed by Maxwell's equations, which, in pH form, read

$$\begin{bmatrix} \varepsilon & 0 \\ 0 & \mu \end{bmatrix} \begin{bmatrix} \dot{\mathbf{E}} \\ \dot{\mathbf{H}} \end{bmatrix} = \left(\begin{bmatrix} 0 & \nabla\times \\ -\nabla\times & 0 \end{bmatrix} - \begin{bmatrix} \sigma & 0 \\ 0 & 0 \end{bmatrix} \right) \begin{bmatrix} \mathbf{E} \\ \mathbf{H} \end{bmatrix} - \begin{bmatrix} \mathbf{J}_0 \\ 0 \end{bmatrix},$$

where $\mathbf{J}_0$ is an impressed current density. The system admits boundary power ports defined by the tangential components $\mathbf{E}_\parallel$, $\mathbf{H}_\parallel$ on the boundary of Ω.

III. Port-Based Coupling in the Discrete Setting

A physically consistent field-line coupling must preserve power-continuity across the interface, retain the combined

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system's Dirac structure without introducing artificial dissipation or loss of reciprocity, and admit a mimetic discretization ensuring discrete energy balance, passivity, and long-term stability. To meet these requirements, we employ compatible staggered spatial discretizations for both subsystems.

A. Discrete pH line subsystem

A staggered finite-difference scheme for the line, which is equivalent to a mixed P_0/P_1 variational discretization, yields the discrete pH system

$$\begin{bmatrix} \mathbf{M}_L & \mathbf{0} \\ \mathbf{0} & \widehat{\mathbf{M}}_C \end{bmatrix} \begin{bmatrix} \dot{\mathbf{I}} \\ \dot{\widehat{\mathbf{V}}} \end{bmatrix} = \left(\begin{bmatrix} \mathbf{0} & \widehat{\mathbf{P}} \\ -\widehat{\mathbf{P}}^\top & \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{M}_R & \mathbf{0} \\ \mathbf{0} & \widehat{\mathbf{M}}_G \end{bmatrix} \right) \begin{bmatrix} \mathbf{I} \\ \widehat{\mathbf{V}} \end{bmatrix},$$

where hats signify interior degrees of freedom, $\mathbf{M}_R$, $\mathbf{M}_L$, $\mathbf{M}_C$, $\mathbf{M}_G$ are the associated diagonal material matrices, $\mathbf{P}$ is the $1/\Delta x$-scaled incidence-based discrete derivative, and boundary inputs are encoded by the port $\mathbf{v} = \mathbf{B}[V_+, V_-]^\top$, where $\mathbf{B}$ consists of the first and last columnn of $\mathbf{P}$.

B. Discrete pH field subsystem

The finite integration technique (FIT) [5] is used for the three-dimensional field subsystem, preserving the exact sequence $\nabla \to \nabla\times \to \nabla\cdot \to 0$ [6]. The resulting discrete pH field subsystem is

$$\begin{bmatrix} \mathbf{M}_\varepsilon & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_\mu \end{bmatrix} \begin{bmatrix} \dot{\mathbf{e}} \\ \dot{\mathbf{h}} \end{bmatrix} = \left(\begin{bmatrix} \mathbf{0} & \mathbf{C} \\ -\mathbf{C}^\top & \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{M}_\sigma & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \right) \begin{bmatrix} \mathbf{e} \\ \mathbf{h} \end{bmatrix} - \begin{bmatrix} \mathbf{J} \\ \mathbf{0} \end{bmatrix},$$

with diagonal material matrices $\mathbf{M}_\varepsilon$, $\mathbf{M}_\mu$, $\mathbf{M}_\sigma$, and discrete curl matrix $\mathbf{C}$. Hence, both discrete subsystems retain their individual pH Dirac structures.

C. Coupled monolithic pH system

By assuming that the line-grid is geometrically aligned with the FIT-grid (union of grid edges), so that each line-voltage gets associated with a primary FIT vertex, it follows that

$$\Delta x \cdot \mathbf{P}\mathbf{V} = -\boldsymbol{\Gamma}\mathbf{e} = -\mathbf{e}_\ell,$$

where the restriction operator $\boldsymbol{\Gamma}$ selects the edge FIT degrees of freedom that correspond to line-voltages. The power exchanged between the line and the field satisfies the discrete Poynting consistency condition $\Delta x \cdot \mathbf{I}^\top \mathbf{P}\mathbf{V} + \mathbf{I}^\top \mathbf{e}_\ell = 0$, ensuring that the interconnection conserves energy across the interface. Note that, with $\mathbf{P} = [\mathbf{e}_1, \widehat{\mathbf{P}}, -\mathbf{e}_m]$, the potential can be reconstructed by discrete integration,

$$\widehat{\mathbf{V}} = V_+\mathbf{A}\mathbf{e}_1 - V_-\mathbf{A}\mathbf{e}_m + (1/\Delta x)\cdot \mathbf{A}\boldsymbol{\Gamma}\mathbf{e},$$

where the matrix $\mathbf{A}$ has entries $A_{ij} = 1$ when $i \geq j$ and zero otherwise (unit lower-trapezoidal matrix) and is the left inverse of $-\Delta x\widehat{\mathbf{P}}$. A monolithic pH system can be obtained by eliminating the duplicate interface variable $\widehat{\mathbf{V}}$ and defining the state vector $\mathbf{x} = [\mathbf{I}; \mathbf{e}; \mathbf{h}]$. This approach yields the dense metric modifications

$$\mathbf{N}_K = \boldsymbol{\Gamma}^\top \mathbf{A}^\top \widehat{\mathbf{M}}_K \mathbf{A}\boldsymbol{\Gamma}, \quad K \in \{C, G\}.$$

More precisely, the resulting global block diagonal material matrix becomes

$$\mathbf{M} = \operatorname{diag}(\mathbf{M}_L, \mathbf{M}_\varepsilon + \mathbf{N}_C, \mathbf{M}_\mu) \succ 0,$$

the dissipation matrix becomes

$$\mathbf{R}_{\mathrm{PH}} = \operatorname{diag}(\mathbf{M}_R, \mathbf{M}_\sigma + \mathbf{N}_G, \mathbf{0}) \succeq 0,$$

and the skew-symmetric interconnection matrix becomes

$$\mathbf{J}_{\mathrm{PH}} = \begin{bmatrix} \mathbf{0} & \widehat{\mathbf{P}}\mathbf{A}\boldsymbol{\Gamma} & \mathbf{0} \\ -(\widehat{\mathbf{P}}\mathbf{A}\boldsymbol{\Gamma})^\top & \mathbf{0} & \mathbf{C} \\ \mathbf{0} & -\mathbf{C}^\top & \mathbf{0} \end{bmatrix}.$$

The external line effort inputs $\mathbf{v}$ modify the field port, so that $\mathbf{u} = [\mathbf{v}; -\mathbf{J}; \mathbf{0}]$ and the coupled monolithic system attains the standard pH form

$$\mathbf{M}\dot{\mathbf{x}} = (\mathbf{J}_{\mathrm{PH}} - \mathbf{R}_{\mathrm{PH}})\,\mathbf{x} + \mathbf{u}.$$

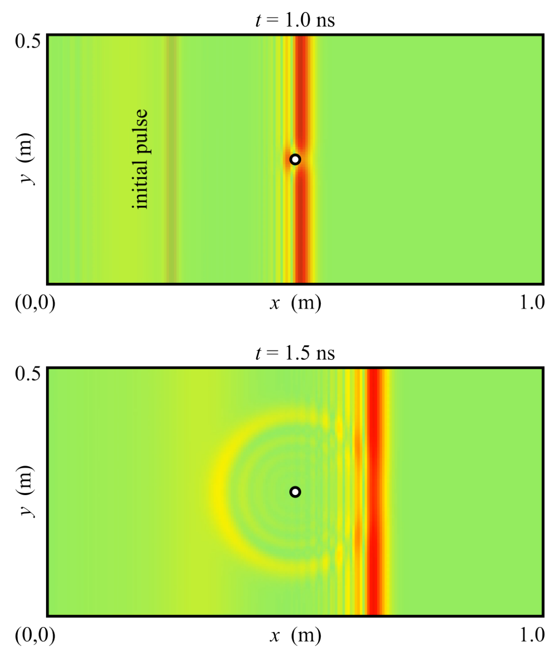

Fig. 2. The magnitude of the electric field strength E when the space is excited by a harmonic pulse at $f = 2.4$ GHz; $R = 0\,\Omega/\mathrm{m}$, $G = 0\,\mathrm{S/m}$, $L = 10^{-8}\,\mathrm{H/m}$, $C = 10^{-8}\,\mathrm{F/m}$.

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