Bergische Universität Wuppertal

Fakultät für Mathematik und Naturwissenschaften

Institute of Mathematical Modelling, Analysis and Computational Mathematics (IMACM)

Preprint BUW-IMACM 25/04

Matthias Ehrhardt, Sergey Pereselkov, Venedikt Kuz'kin, Sergey Tkachenko

# Underwater Sound Source Depth Estimation by Single Receiver in Shallow Water

March 2, 2025

http://www.imacm.uni-wuppertal.de

# Underwater Sound Source Depth Estimation by Single Receiver in Shallow Water

Matthias Ehrhardt, Sergey Pereselkov, Venedikt Kuz'kin, Sergey Tkachenko

***Abstract*—The paper proposes a passive acoustic method of remote sensing in shallow water. The method allows to estimate the depth of a sound source using a single receiver. The depth estimation is based on the amplitude ratio data of the acoustic modes of the sound source field at the receiver point. The problem statement and the proposed method are described. The efficiency of the method is analyzed. It is shown that the error in the depth estimation is bounded by the structure of the acoustic modes. The results of applying the method to numerical modeling are presented. The method is verified using data from the Yellow Sea experiment (2004). The results of numerical simulation and experimental verification are in agreement with the theoretical assumptions and confirm the effectiveness of the proposed method.**

***Index Terms*—Remote sensing, shallow water, passive method, sound source, depth estimation, single receiver, acoustic modes**

## I. Introduction

THE passive acoustic methods of remote sensing for localization of underwater sound source, in a shallow water are a topic of great interest. At present, passive acoustic methods based on matched field processing (MFP) [1], [2] and matched mode processing (MMP) [3], [4] are most commonly used for the localization of underwater sound sources in shallow water. These MFP and MMP methods require detailed knowledge of the propagation environment (water layer and bottom parameters) and a vertical line array (VLA) [5].

The main drawbacks of these methods are (i) their sensitivity to discrepancies between the model and the actual waveguide properties [6], and (ii) their susceptibility to noise, which can significantly reduce the accuracy of the results [7]. This problem is particularly important in practical applications. It is especially critical in shallow water environments, where there is a lack of reliable data on bottom characteristics, and where time-varying processes in the water column can lead to significant hydrological fluctuations, further complicating the problem. Source localization by MFP and MMP typically involves a simultaneous search for both the range and depth of the source using the range-depth ambiguity function.

Often, an inaccurate estimate of one parameter (e.g., depth) results from an inaccurate estimate of the other parameter (e.g., range), especially in the presence of high sidelobes. The sidelobe problem becomes more pronounced when the assumed environment differs from the actual one, a situation known as the environmental mismatch problem. Estimating the depth or range of the source is generally not reliable if the estimation of one depends on the other.

For some applications, an approximate estimate of the source depth is sufficient, even if the source range is not known. This is known as the underwater source depth classification (discrimination) problem in remote sensing. There are a variety of approaches to solving the source depth estimation problem [8]: reliable acoustic path [9], arrival angle [9], and time delay difference [10], and methods such as correlation matching [11], [12] and frequency-dependent matching [13]–[15]. Most of these depth estimation methods share common drawbacks, such as complex theoretical derivations, extensive computations, dependence on specific signal forms (as in time-reversal methods), or the need for a high signal-to-noise ratio, among others.

The aim of this paper is to propose a passive acoustic method for remote sensing in shallow water. The proposed method is simple and free from the mentioned drawbacks. The method allows to estimate the depth of a sound source using a single receiver. The depth estimation is based on the amplitude ratio data of the acoustic modes of the sound source field at the receiver point. The problem statement and the proposed method are described. The efficiency of the method is analyzed. It is shown that the error in the depth estimation is bounded by the structure of the acoustic modes. The results of applying the method to numerical modeling are presented. The method is verified using data from the Yellow Sea experiment. The results of numerical simulation and experimental verification are in agreement with the theoretical assumptions and confirm the effectiveness of the proposed method.

The paper is divided into seven sections. Section I provides a comprehensive review of the papers on passive methods of sound source depth estimation. Section II deals with the problem statement of a method that allows to estimate the depth of a sound source using a single receiver. Section III describes the proposed method, which is based on the amplitude ratio data of the acoustic modes of the sound source field at the receiver point. Section IV contains the results of the application of the method in the framework of numerical simulation. Section V presents the results of verification of the proposed method using data from the Yellow Sea experiment. Finally, Section VI summarizes the results of the research, highlighting the main findings and contributions in passive acoustic method of remote sensing in shallow water.

M. Ehrhardt is with the Chair of Applied and Computational Mathematics, University of Wuppertal, Wuppertal, Germany (e-mail: ehrhardt@uni-wuppertal.de).

S. Pereselkov is with the Mathematical Physics and Information Technology Department, Voronezh State University, Voronezh, Russia (e-mail: pereselkov@yandex.ru).

V. Kuz'kin is with the Prokhorov General Physics Institute of the Russian Academy of Sciences, Moscow, Russia (e-mail: kumiov@yandex.ru).

S. Tkachenko is with the Mathematical Physics and Information Technology Department, Voronezh State University, Voronezh, Russia (e-mail: tkachenko.edu@yandex.ru).

## II. Problem Statement

Consider a shallow water waveguide (Fig. 1) of depth $H$ in coordinates $(r, z)$. The signal emitted by the point source $S$ at frequency $\omega = 2\pi f$ at depth $z_s$ is received by the point receiver $Q$ at depth $z_q$. The source $S$ is placed at a horizontal distance $r$ from the receiver $Q$.

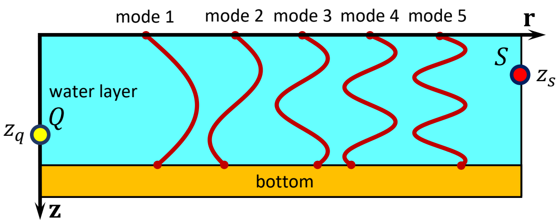


Fig. 1. Problem statement. Shallow water waveguide model.

The sound field of a source $S$ at receiver point $Q$ is represented as the sum of the propagating modes [16]:

$$\begin{aligned} p(r, z_s, z_q, \omega) &= \sum_{m=1}^{M} p_m(r, z_s, z_q, \omega) \\ &= \sum_{m=1}^{M} A_m(r, z_s, z_q, \omega)\, \exp\big[\mathrm{i}h_m(\omega)r\big], \end{aligned} \tag{1}$$

where $A_m$ and $h_m$ are the amplitude and wavenumber of the mode $\phi_m(z)$ with number $m$. Here $M$ is the number of propagating modes.

Let us consider a time dependence $\zeta(\tau)$ of the received signal from a source in the frequency band:

$$\omega_0 - \Delta\omega/2 \leq \omega \leq \omega_0 + \Delta\omega/2. \tag{2}$$

To define $\zeta(\tau)$, we apply the Fourier transform (1) in the frequency band (2):

$$\begin{aligned} \zeta(\tau) &= \int_{\omega_0-\Delta\omega/2}^{\omega_0+\Delta\omega/2} p(r, z_s, z_q, \omega)\, \exp(-\mathrm{i}\omega\tau)\, d\omega \\ &= \sum_{m=1}^{N} \zeta_m(\tau), \end{aligned} \tag{3}$$

where $\zeta_m(\tau)$ is the time dependence of the signal of mode $\phi_m(z)$ with number $m$

$$\zeta_m(\tau) = \int_{\omega_0-\Delta\omega/2}^{\omega_0+\Delta\omega/2} p_m(r, z_s, z_q, \omega) \exp(-\mathrm{i}\omega\tau)\, d\omega. \tag{4}$$

We consider the amplitude $A_m$ as a slowly changing function compared to the phase, which makes it possible to remove it from the integral for a value of $\omega = \omega_0$. As a result, we obtain the following expression for the time dependence of the mode $\zeta_m(\tau)$:

$$\begin{aligned} \zeta_m(\tau) = A_m(r, z_s, z_q, \omega_0)\Delta\omega \exp\big\{\mathrm{i}[h_m(\omega_0)r - \omega_0\tau]\big\} \\ \times \frac{\sin\big[\Delta\omega(\tau-\tau_m)/2\big]}{[\Delta\omega(\tau-\tau_m)/2]}. \end{aligned} \tag{5}$$

According to Equation (5), the position of the main maximum of the mode signal $\zeta_m(\tau)$ has a value of $\tau = \tau_m$. Here $\tau_m = r/u_m(f_0)$ is the propagation time of the $m$th mode and $u_m = d(\omega)/dh_m$ is the group velocity. The expression (5) is valid if the following conditions are satisfied.

1) The dispersion $h_m(\omega)$ is linear:

$$|\omega - \omega_0| \ll 2\left|\frac{dh_m(\omega)}{d\omega}\Big/\frac{d^2h_m(\omega)}{d\omega^2}\right|, \tag{6}$$

2) The amplitude $A_m(\omega)$ is a slowly changing:

$$\left|\frac{\partial A_m(\omega)}{\partial\omega}\right| \ll \left|A_m(\omega)r\frac{dh_m(\omega)}{d\omega}\right|, \tag{7}$$

The conditions (6) and (7) mean that the mode signal $\zeta_m(\tau)$ can be considered as a quasi-monochromatic wave.

It can be seen that $\zeta(\tau)$ is the sum of the modes signals $\zeta_m(\tau)$. Let us use the Rayleigh criterion [17] of resolvability of modes signals, as follows from (5). The neighboring mode signals are isolated if the following condition is satisfied:

$$\tau_{m+1} - \tau_m \geq \frac{2\pi}{\Delta\omega}. \tag{8}$$

When resolving modes signals, the modulus of the expression for the signal from the source has the form:

$$|\zeta(\tau)| \approx \sum_{m=1}^{M} |\zeta_m(\tau)|, \tag{9}$$

i.e., the envelope of the received signal is approximately equal to the sum of the envelopes of the mode pulses.

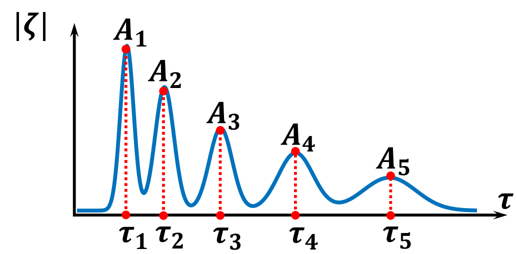


Fig. 2. Problem statement. Time dependence $|\zeta(\tau)|$ of the received signal.

In Fig. 2 the time dependence $|\zeta(\tau)|$ of the received signal from a source described by (5) is shown, if the condition (8) is fulfilled. In this case it is possible to estimate the value of the amplitudes $A_m$ of the mode signals $|\zeta_m(\tau)|$. These values of the mode amplitudes $A_m$, determined from the received signal $|\zeta(\tau)|$, are used as input data within the framework of the proposed method for underwater source depth estimation.

## III. Proposed Method

Consider the ratio of the amplitudes of two neighboring modes $A_m(r, z_s, z_q, f)$ and $A_n(r, z_s, z_q, f)$:

$$\chi_{mn}^{(ex)}(z_s, z_q, f) = A_m(r, z_s, z_q, f)/A_n(r, z_s, z_q, f). \tag{10}$$

According to the mode description of the sound field [16], the experimental value of the ratio $\chi_{mn}^{(ex)}$ is equal to:

$$\chi_{mn}^{(ex)}(z_s, z_q, f) = \left|\frac{\phi_m^{(ex)}(z_s, f)\,\phi_m^{(ex)}(z_q, f)}{\phi_n^{(ex)}(z_s, f)\,\phi_n^{(ex)}(z_q, f)}\right|. \tag{11}$$

Here '$(ex)$' means the experimental result. In the mode description of the source field, the information about its source depth $z_s$ is contained in the modes $\phi_m(z_s, f)$, where $f$ is the reference frequency of the considered frequency band. The depth $z$ of the source is estimated by solving the equation:

$$\frac{\phi_m^{(ex)}(z,f)}{\phi_n^{(ex)}(z,f)} = \chi_{mn}^{(ex)}(z_s,z_q,f)\,\frac{\phi_n^{(ex)}(z_q,f)}{\phi_m^{(ex)}(z_q,f)}. \qquad (12)$$

The equation (12) represents the main idea of the proposed method.

Let us introduce the function $\Theta_{mn}(z)$ as the difference between theoretical $\chi_{mn}^{(th)}$ and experimental $\chi_{mn}^{(ex)}$ values:

$$\Theta_{mn}(z) = \left|\chi_{mn}^{(th)}(z,z_q,f) - \chi_{mn}^{(ex)}(z_s,z_q,f)\right|, \qquad (13)$$

where

$$\chi_{mn}^{(th)}(z,z_q,f) = \frac{\phi_m^{(th)}(z,f)\,\phi_m^{(th)}(z_q,f)}{\phi_n^{(th)}(z,f)\,\phi_n^{(th)}(z_q,f)}. \qquad (14)$$

The proposed method is based on the equations (10)–(14). The estimation of the source depth is the value $z = z'$, for which the expression (13) becomes zero:

$$\Theta_{mn}(z') = 0. \qquad (15)$$

However, the solution is multi-valued, meaning that there are multiple possible values of $z'$ for a given pair of modes $\phi_m(z)$ and $\phi_n(z)$. To resolve this ambiguity and determine the true value, depth estimation should be performed using multiple mode pairs or different frequency bands for a given mode pair. For different mode pairs, the solution set of (15) contains only the correct matching solution, which provides an accurate estimate of the source depth. All other non-matching solutions are incorrect estimates. Thus, the estimated source depth should be chosen as the value $z_s^* = z'$ that is common to all mode pairs.

## IV. Numerical Simulation

Let us consider the results of the source depth estimation within the framework of the numerical simulation of the received signal in the shallow water waveguide. The results of numerical simulations are shown in Figs. 3 – 6.

We assumed that the parameters of the numerical modeling are similar to the parameters of the experimental waveguide of Section V. Depth of the water layer $H = 53\,\mathrm{m}$. Sound speed of water layer is constant $c = 1474\,\mathrm{ms}^{-1}$. Sound speed of the bottom layer is $c = 1700\,\mathrm{ms}^{-1}$. Density of the bottom layer is $\rho = 1.8\,\mathrm{gcm}^{-3}$. Bottom absorption parameter is $\alpha = 0.01$. Distance between source and receiver is $r = 10\,\mathrm{km}$. The received signal is considered in the frequency band $\Delta f = 120 - 160\,\mathrm{Hz}$. The modes $\phi_m(z)$ are determined by numerical solutions of the Sturm-Liouville problem with corresponding boundary conditions. The receiver depth is $z_q = 52\,\mathrm{m}$. Two cases of source depth are considered: $z_s = 15\,\mathrm{m}$ and $z_s = 28\,\mathrm{m}$. The results in Figs. 3 and 4 correspond to $z_s = 15\,\mathrm{m}$. The results in Figs. 5 and 6 correspond to $z_s = 28\,\mathrm{m}$.

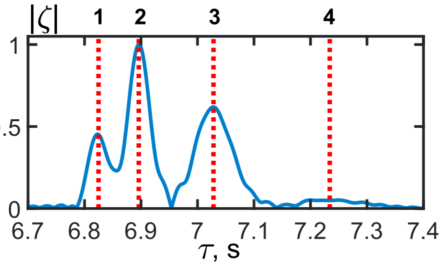


Fig. 3. Numerical simulation. Time dependence $|\zeta(\tau)|$ of the received signal, cf. (3). $z_s = 15\,\mathrm{m}$, $z_q = 52\,\mathrm{m}$. $\Delta f = 120 - 160\,\mathrm{Hz}$.

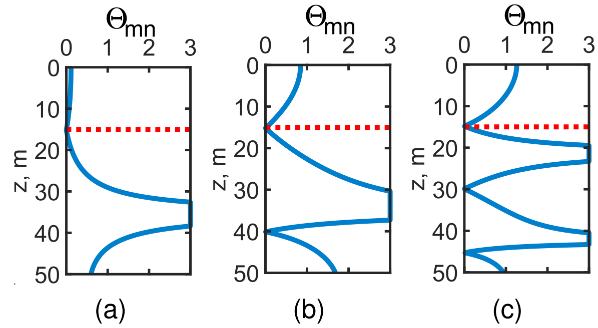


Fig. 4. Numerical simulation of $\Theta_{mn}(z)$. $z_s = 15\,\mathrm{m}$, $z_q = 52\,\mathrm{m}$. $\Delta f = 120 - 160\,\mathrm{Hz}$. (a) $m = 1$, $n = 2$. (b) $m = 3$, $n = 2$. (c) $m = 4$, $n = 3$.

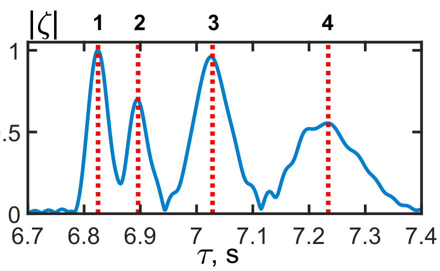


Fig. 5. Numerical simulation. Time dependence $\zeta(\tau)$ of the received signal. $z_s = 28\,\mathrm{m}$, $z_q = 52\,\mathrm{m}$. $\Delta f = 120 - 160\,\mathrm{Hz}$.

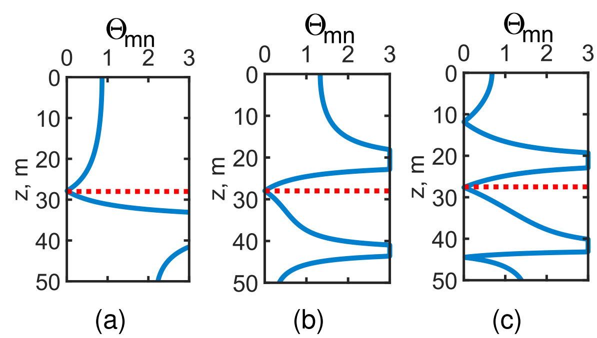


Fig. 6. Numerical simulation of $\Theta_{mn}(z)$. $z_s = 28\,\mathrm{m}$, $z_q = 52\,\mathrm{m}$. $\Delta f = 120 - 160\,\mathrm{Hz}$. (a) $m = 1$, $n = 2$. (b) $m = 3$, $n = 2$. (c) $m = 4$, $n = 3$.

The normalized time dependence of $|\zeta(\tau)|$ of the received signal for case: $z_s = 15\,\mathrm{m}$ is shown in Fig. 3. The received signal contains four modes: $A_1 = 0.54$, $A_2 = 1.0$, $A_3 = 0.6$, $A_4 = 0.05$. The function $\Theta_{mn}(z)$ calculated for these values of $A_m$ (Eq. (13)) is shown in Fig. 4 for different pairs of modes: (a) $m = 1$, $n = 2$. (b) $m = 3$, $n = 2$. (c) $m = 4$, $n = 3$. You can see that $\Theta_{12}(z)$ has only one zero at $z = 15\,\mathrm{m}$. $\Theta_{32}(z)$ has two zeros at $z = 15, 40\,\mathrm{m}$. $\Theta_{43}(z)$ has tree zeros at $z = 15, 30, 45\,\mathrm{m}$. The estimated source depth is a common value for all mode pairs $z_s^* = 15\,\mathrm{m}$. The normalized time dependence of $|\zeta(\tau)|$ of the received signal for another case: $z_s = 28\,\mathrm{m}$ is shown in Fig. 5. The signal consists of four modes signals $|\zeta_m(\tau)|$: $A_1 = 1.0$, $A_2 = 0.7$, $A_3 = 0.96$, $A_4 = 0.55$. The function $\Theta_{mn}(z)$ calculated for these values of $A_m$ is shown in Fig. 6 for different pairs of modes: (a) $m = 1$, $n = 2$. (b) $m = 3$, $n = 2$. (c) $m = 4$, $n = 3$. It can be seen that $\Theta_{12}(z)$ has only one zero at $z = 28\,\mathrm{m}$. In contrast to the previous case in Fig. 4(b), $\Theta_{32}(z)$ has only one zero at $z = 28\,\mathrm{m}$. $\Theta_{43}(z)$ has tree zeros at $z = 11, 28, 44\,\mathrm{m}$. As a result, the common value for all mode pairs is $z_s^* = 28\,\mathrm{m}$. This is the estimated source depth for the second case of the numerical simulation. It should be noted that in the numerical simulation we obtained accurate values for the source depth in both cases ($z_s^* = 15\,\mathrm{m}$ and $z_s^* = 28\,\mathrm{m}$), since we assumed that "experimental" and "theoretical" modes are the same: $\phi_n^{(th)}(z) = \phi_n^{(ex)}(z) = \phi_n(z)$. In practice, the experimental modes may be known only approximately: $\phi_n^{(th)}(z) \approx \phi_n^{(ex)}(z)$. Therefore, the depth estimate ($z_s^*$) obtained in the experiment may differ from the true value of the source depth ($z_s$).

## V. Experimental Results

We consider the experimental verification of the proposed source depth estimation method in real shallow water conditions. We briefly describe the experimental setting. The experiment was conducted in 2004 on the Pacific Shelf (Yellow Sea). The depth of the water layer is $H = 53\,\mathrm{m}$. The experimental data on water layer stratification and bottom parameters are close to those described in Section IV. The receiver depth is $z_q = 52\,\mathrm{m}$. The airgun source was used as a broadband sound source and the sound source with depth $z_s \approx 15\,\mathrm{m}$ was towed by a research vessel with speed $v \approx 1.7\,\mathrm{ms}^{-1}$. The airgun source produced broadband pulses, separated by a time interval of $T = 30\,\mathrm{s}$, which consistently exhibited repeatable spectra in the range of $\Delta f \approx 0 - 250\,\mathrm{Hz}$. As part of the experimental verification, we use the airgun pulse when the distance between the source and receiver is $r = 10\,\mathrm{km}$.

The received signal is shown in Fig. 7. The normalized time dependence of the received signal $\zeta(\tau)$, in the frequency band $\Delta f = 0 - 250\,\mathrm{Hz}$ is shown in Fig. 7(a). The normalized spectrogram of the received signal, in the frequency band $\Delta f = 0 - 200\,\mathrm{Hz}$ is shown in Fig. 7(b). The normalized time dependence of $|\zeta(\tau)|$, $\Delta f = 120 - 160\,\mathrm{Hz}$ is shown in Fig. 7(c). It can be seen that the experimental $|\zeta(\tau)|$ is close to the modeling signal Fig. 3 in Section IV. The received signal contains four modes: $A_1 = 0.57$, $A_2 = 1.0$, $A_3 = 0.54$, $A_4 = 0.07$.

The task of determining the theoretical modes $\phi_n^{(th)}(z) = \phi_n(z)$ is solved as in Section IV by using numerical solutions of the Sturm-Liouville problem with corresponding boundary conditions. In this case, however, we know the theoretical modes only approximately: $\phi_n^{(th)}(z) \approx \phi_n^{(ex)}(z)$. So the function $\Theta_{mn}(z)$ is calculated approximately.

In Fig. 8 the function $\Theta_{mn}(z)$ for the mentioned values of $A_m$ (Eq. (13)) is shown for different pairs of modes: (a) $m = 1$, $n = 2$. (b) $m = 3$, $n = 2$. (c) $m = 4$, $n = 3$. It can be seen that $\Theta_{12}(z)$ has only one zero at $z = 16.5\,\mathrm{m}$. $\Theta_{32}(z)$ has two zeros at $z = 16.0, 41\,\mathrm{m}$. $\Theta_{43}(z)$ has tree zeros at $z = 15, 30, 45\,\mathrm{m}$. The estimated source depth is a common value for all mode pairs $z_s^* \approx 15.8\,\mathrm{m}$. It can be seen that the depth estimate ($z_s^* \approx 15.8\,\mathrm{m}$) obtained in the considered experiment may differ from the true value of the source depth ($z_s = 15\,\mathrm{m}$). The difference is less than 1 meter. The reason for this discrepancy is the difference in the mode functions $\phi_n^{(th)}(z) \approx \phi_n^{(ex)}(z)$.

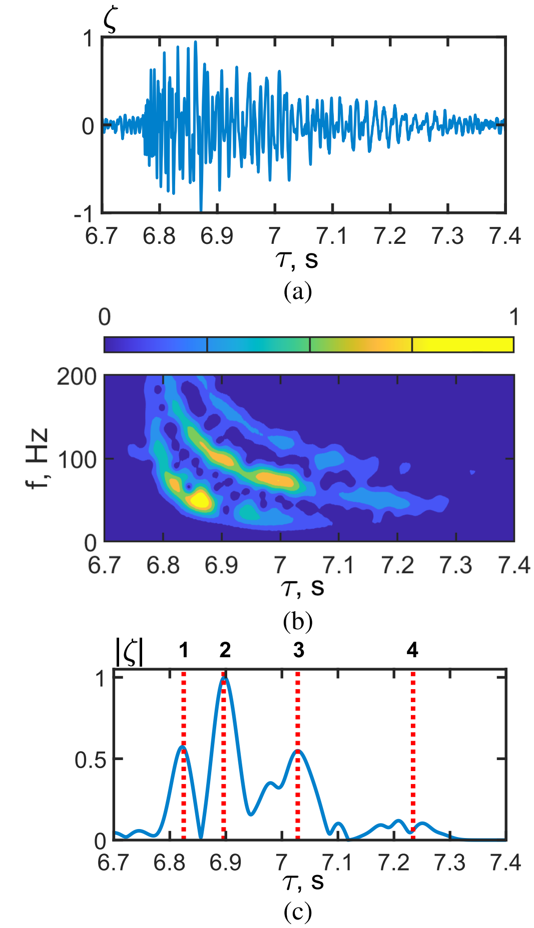


Fig. 7. Experimental results. (a) Normalized time dependence of received signal $\zeta(\tau)$, $\Delta f = 0 - 250\,\mathrm{Hz}$. (b) Normalized spectrogram of received signal, $\Delta f = 0 - 200\,\mathrm{Hz}$. (c) Normalized time dependence of $|\zeta(\tau)|$, $\Delta f = 120 - 160\,\mathrm{Hz}$.

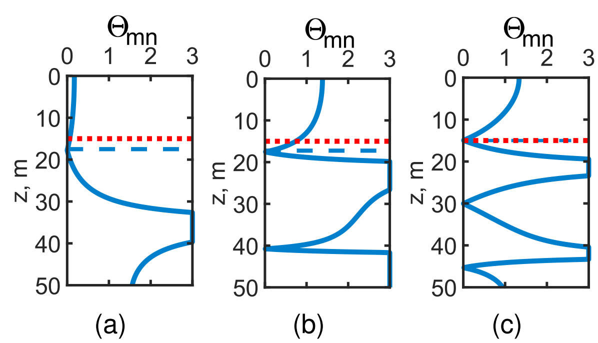


Fig. 8. Experimental results. Function $\Theta_{mn}(z)$. $\Delta f = 120 - 160\,\mathrm{Hz}$. (a) $m = 1$, $n = 2$. (b) $m = 3$, $n = 2$. (c) $m = 4$, $n = 3$.

It should be noted that the estimation of the source depth becomes more accurate as we move from pairs of lower modes to pairs of higher modes. Thus, it can be stated that the proposed source depth estimation method will provide a more accurate estimate as the frequency range increases. At the same time, an increase in frequency will lead to a denser filling of the mode set. This, in turn, will make it more difficult to separate the modes, which is necessary to determine the amplitudes of the mode signals. Thus, it can be concluded that there is an optimal frequency range for the given distance, in which the use of the proposed source depth estimation method is most effective.

## VI. Conclusion

The paper proposes a passive acoustic remote sensing technique designed for shallow water environments. This method allows the estimation of the depth of a sound source using only a single hydrophone. The approach is based on the analysis of the amplitude ratios of the acoustic modes present in the sound field generated by the source at the receiver location. The amplitude ratios of these modes serve as key indicators for estimating the source depth. By exploiting the modal structure of the acoustic field, the method provides a way to infer the source depth without the need for complex array configurations or active sensing systems.

The efficiency and accuracy of the method are thoroughly analyzed. It is shown that the error in depth estimation is inherently limited by the structure of the acoustic modes, which are determined by the environmental parameters and the frequency band.

The simulation results are presented, showing that the method performs well under different environmental configurations: source depth values. In addition, the method is tested using experimental data collected during the Yellow Sea experiment (2004). The experimental results are close to the true value (mismatch less than 1 m), confirming the effectiveness and robustness of the method under real-world conditions.

In conclusion, the proposed passive acoustic method offers a promising solution for remote depth estimation in shallow water environments for applications in underwater acoustics, oceanography, and marine resource exploration. The agreement between numerical, experimental, and theoretical results underscores the reliability and potential of this approach for practical use.

In our future work, we plan to perform a more thorough experimental verification of the proposed source depth estimation method. In addition, we intend to develop a method that combines the approach described in this paper with the holographic signal processing proposed in our previous work [18], [19].

## Acknowledgments

This research was supported by a grant from the Russian Science Foundation No. 23-61-10024, https://rscf.ru/project/23-61-10024/.

## References

[1] A. B. Baggeroer, W. A. Kuperman, and P. N. Mikhalevsky, “An overview of matched field methods in ocean acoustics,” *IEEE J. Oceanic Eng.*, vol. 18, pp. 401–424, 1993.

[2] A. Tolstoy, *Matched Field Processing for Underwater Acoustics*, World Scientific, Singapore, 1993, pp. 1–212.

[3] T. C. Yang, “A method of range and depth estimation by modal decomposition,” *J. Acoust. Soc. Am.*, vol. 82, pp. 1736–1745, 1987.

[4] E. C. Shang, “Source depth estimation in waveguides,” *J. Acoust. Soc. Am.*, vol. 77, pp. 1413–1418, 1985.

[5] C. W. Bogart and T. C. Yang, “Matched field localization with horizontal arrays in shallow water; spatial sampling and effective aperture,” *J. Acoust. Soc. Am.*, vol. 96, pp. 1677–1686, 1994.

[6] D. Gingrass, “Methods for Predicting the Sensitivity of Matched-Field Processors to Mismatch,” *J. Acoust. Soc. Am.*, vol. 86, p. 1940, 1989.

[7] J. L. Krolik, “Matched-Field Minimum Variance Beamforming in a Random Ocean Channel,” *J. Acoust. Soc. Am.*, vol. 92, no. 3, p. 1408, 1992.

[8] K. Yang, Q. Yang, X. Guo, and R. Cao, “A simple method for source depth estimation with multipath time delay in the deep ocean,” *Chinese Phys. Lett.*, vol. 33, no. 12, p. 124302, 2016.

[9] R. Duan, K. Yang, Y. Ma, and B. Lei, “A reliable acoustic path: Physical properties and a source localization method,” *Chinese Phys. B*, vol. 21, no. 12, p. 124301, 2012.

[10] K. Yang, Y. Lu, Z. Lei, and H. Xia, “Passive localization based on multipath time-delay difference with two hydrophones in deep ocean,” *Acoust. Australia*, vol. 45, no. 1, pp. 51—60, 2017.

[11] Z. Lei, K. Yang, and Y. Ma, “Passive localization in the deep ocean based on the cross-correlation function matching,” *J. Acoust. Soc. Am.*, vol. 139, no. 6, pp. EL196—EL201, 2016.

[12] K. Yang, H. Li, R. Duan, and Q. Yang, “Analysis on the characteristic of cross-correlated field and its potential application on source localization in deep water,” *J. Comput. Acoust.*, vol. 25, no. 2, p. 1750001, 2017.

[13] B. M. Worthmann and D. R. Dowling, “High frequency source localization in a shallow ocean sound channel using frequency-difference matched field processing,” *J. Acoust. Soc. Am.*, vol. 138, no. 6, pp. 3549—3562, 2015.

[14] P. Hursky, M. B. Porter, M. Siderius, and V. K. Mcdonald, “High-frequency (8-16 kHz) model-based source localization,” *J. Acoust. Soc. Am.*, vol. 115, no. 6, pp. 3021—3032, 2004.

[15] S. P. Czenszak and J. L. Krolik, “Robust wideband matched-field processing with a short vertical array,” *J. Acoust. Soc. Am.*, vol. 101, no. 2, pp. 749—759, 1997.

[16] F. B. Jensen, W. A. Kuperman, M. B. Porter, and H. Schmidt, *Computational Ocean Acoustics*, Springer, Berlin/Heidelberg, Germany, 2011.

[17] V. M. Kuz’kin, M. V. Kutsov, and S. A. Pereselkov, “Frequency shifts initiated by surface roughness,” *Phys. Wave Phenom.*, vol. 22, pp. 144–149, 2014.

[18] S. A. Pereselkov and V. M. Kuz’kin, “Interferometric processing of hydroacoustic signals for the purpose of source localization,” *J. Acoust. Soc. Am.*, vol. 151, pp. 666–676, 2022.

[19] M. Ehrhardt, S. A. Pereselkov, V. M. Kuz’kin, I. Kaznacheev, and P. Rybyanets, “Experimental observation and theoretical analysis of the low-frequency source interferogram and hologram in shallow water,” *J. Sound Vibr.*, vol. 544, p. 117388, 2023.